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NRL Report 8105

TE- Mode Solutions for Dielectric Slab Center Loaded Ridged Waveguide

CHARLES W. YOUNG, JR.

*Microwave Techniques Branch
Electronics Technology Division*

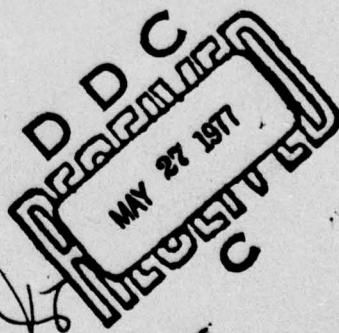
April 29, 1977



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TE-MODE SOLUTIONS FOR DIELECTRIC-SLAB CENTER-LOADED RIDGED WAVEGUIDE

INTRODUCTION

Applications exist which require dielectric or ferrite slab center loaded rectangular waveguide to be used in conjunction with ridged waveguide. Transitions can be made with stepped matching transformers; these transformers are appropriate sections of a composite of the two different waveguide configurations. The objective of this report is to present an analysis of such dielectric slab center loaded ridged waveguide and to provide a method of obtaining equations for the TE_{n0} propagation characteristics, thus facilitating transformer design.

BACKGROUND

Because higher order modes can easily cause mismatch and transmission loss spikes, waveguide operation is generally limited to a frequency band where only the principal mode may propagate. Conventional rectangular waveguide has a theoretical two-to-one, or single octave, principal-mode-only frequency bandwidth; in practice, the useable bandwidth is less because of large attenuation near the cut-off frequency.

Ridged waveguide, particularly double-ridged waveguide, is commonly used when larger bandwidths are required at high power levels. A frequency range of more than four to one between the cut-off frequencies of the TE_{10} and TE_{20} modes can easily be obtained [1,2] using double-ridged waveguide. Similar bandwidths can be obtained with dielectric slab center loaded rectangular waveguide [3,4,5]. Both ridged and slab loaded waveguide achieve broad bandwidths by adding large capacitance to the dominant mode while only slightly affecting the capacitance of the next higher order mode.

Ferrite toroidal phase shifters also can be designed for operation in excess of one octave. Because of the small gap spacing of ridged waveguide, the phase shifters are generally made in rectangular waveguide. Dielectric slab center loaded rectangular waveguide would be readily compatible with the ferrite toroidal phase shifter, but it is not a commonly used transmission line. Since ridged waveguide is commonly used, it would be desirable to have compatibility, i.e., matching transitions, between ridged waveguide and ferrite toroidal loaded rectangular waveguide. Dielectric loaded tapered transitions are possible, but the fabrication would be very difficult. Also, a quasi-Tchebycheff transformer design should give better matching for given length transitions. The latter approach requires transformer sections of dielectric loaded ridged waveguide, but analysis of this type of transmission line is not currently available in the literature. The analysis in this report employs an equivalent transmission line circuit for the transverse component of the propagating electromagnetic wave to derive solutions for the TE_{n0} propagation constants in such waveguide.

ANALYSIS

TE-mode solutions for the dielectric slab center loaded rectangular waveguide of Fig. 1a can be derived by using ABCD matrices [6] or by using an equivalent transmission line circuit for the crossguide component of the electromagnetic wave. The homogeneous double-ridged waveguide of Fig. 1b has been analyzed [1,2] by using the latter method in conjunction with the equivalent discontinuity susceptance due to the height change at the ridge wall.

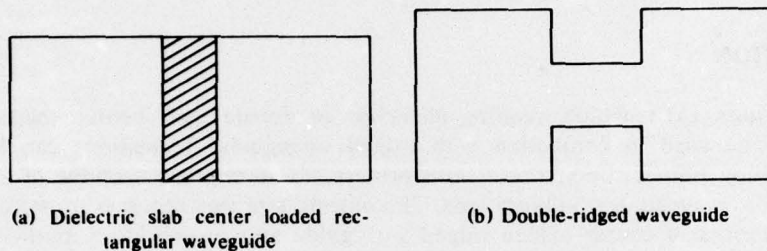


Fig. 1 — Broadband waveguide cross sections

For the dielectric slab center loaded double ridged waveguide of Fig. 2, the analysis is similar to that for the homogeneous case, with an extra section incorporated in the equivalent transmission line circuit. The dimensions referred to in all subsequent calculations are those shown in Fig. 2. For simplicity, this report will consider only the case for TE_{n0} modes and will assume that the transmission line is lossless, i.e., perfectly conducting waveguide walls and a dielectric loss tangent of zero. Axial symmetry will also be assumed.

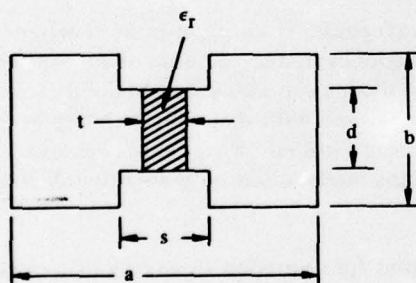
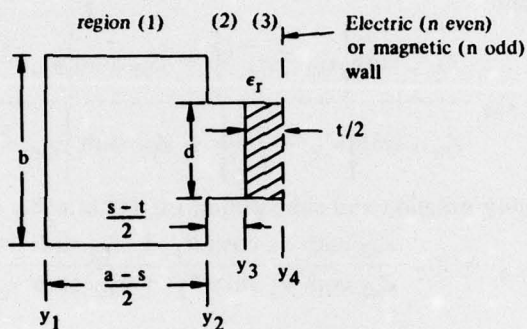


Fig. 2 — Cross section of dielectric slab center loaded, double-ridged waveguide

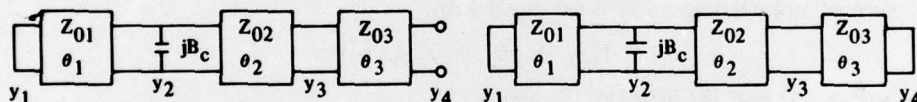
Cohn's article on ridged waveguide [2] points out that for the homogeneous case (i.e. $\epsilon_r = 1$) the cross section may be treated at the cut-off frequency by assuming that it is an infinitely wide, composite, parallel strip transmission line short-circuited at two points. The resultant electromagnetic field may be considered as an electromagnetic wave traveling from side to side without longitudinal propagation. The resonant conditions can then be solved for the cut-off frequencies of the different TE_{n0} modes.

A similar argument holds for the inhomogeneous case. In addition, the longitudinal propagation constant may be treated as the unknown quantity, and solutions at any frequency may be obtained by separating the wave vector in each region into its transverse and longitudinal components. Since the waveguide configuration is symmetrical, the resonance condition for the transverse wave component will result in an infinite (zero) impedance at the center for n odd (even). Half of a cross section is shown in Fig. 3a, and the equivalent transmission line circuits

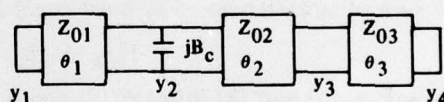
for the transverse wave are shown in Fig. 3b for n odd and Fig. 3c for n even. Since the equivalent circuit is a composite, dissipationless, passive line matched at both ends, it is matched at all points. Therefore, the sum of the admittances at the plane y_2 of the effective lumped capacitance due to the ridge wall must equal zero. Within each region, where the regions are shown in Fig. 3a, Z_{0i} is the characteristic impedance, $Y_{0i} = 1/Z_{0i}$ is the characteristic admittance, and θ_i is the transverse electrical length; θ_i is equal to the product of the physical transverse dimension of the region and γ_{yi} , the complex transverse propagation constant. Since all regions are lossless, γ_{yi} , and therefore θ_i , will be purely real or purely imaginary.



(a) Half cross section



(b) Equivalent transmission line circuit for n odd



(c) Equivalent transmission line circuit for n even

Fig. 3 — Half waveguide cross section and equivalent transmission line circuits for transverse wave

The reflected impedance Z presented by a load impedance Z_L terminating a transmission line of characteristic impedance Z_0 with propagation constant γ and length w is [7]

$$Z = Z_0 \frac{(Z_L + Z_0)e^{\gamma w} + (Z_L - Z_0)e^{-\gamma w}}{(Z_L + Z_0)e^{\gamma w} - (Z_L - Z_0)e^{-\gamma w}} \quad (1)$$

The short circuit at y_1 in Fig. 3b will be reflected back to y_2 as Z_{1-2} where

$$Z_{1-2} = Z_{01} \tanh \left[\gamma_{y1} \frac{a-s}{2} \right] \quad (2)$$

or

$$Y_{1-2} = Y_{01} \coth \left[\gamma_{y1} \frac{a-s}{2} \right] \quad (3)$$

The open circuit at y_4 will reflect back to y_3 as Z_{4-3} with

$$Z_{4-3} = Z_{03} \coth \left[\gamma_{y3} \frac{l}{2} \right]. \quad (4)$$

Equation (1) can be rewritten in the form

$$Z = Z_0 \frac{Z_L \cosh \gamma w + Z_0 \sinh \gamma w}{Z_L \sinh \gamma w + Z_0 \cosh \gamma w}. \quad (5)$$

Since Z_{4-3} terminates region 2,

$$Z_{4-2} = Z_{02} \frac{Z_{4-3} \cosh \left[\gamma_{y2} \frac{s-l}{2} \right] + Z_{02} \sinh \left[\gamma_{y2} \frac{s-l}{2} \right]}{Z_{4-3} \sinh \left[\gamma_{y2} \frac{s-l}{2} \right] + Z_{02} \cosh \left[\gamma_{y2} \frac{s-l}{2} \right]}. \quad (6)$$

Using $\theta_i = \gamma_{yi} w_i$ to simplify notation and substituting Eq. (4) into Eq. (6) yields

$$Z_{4-2} = Z_{02} \frac{Z_{03} \coth \theta_3 \cosh \theta_2 + Z_{02} \sinh \theta_2}{Z_{03} \coth \theta_3 \sinh \theta_2 + Z_{02} \cosh \theta_2} \quad (7)$$

or

$$Y_{4-2} = Y_{02} \frac{Z_{03} \coth \theta_3 \sinh \theta_2 + Z_{02} \cosh \theta_2}{Z_{03} \coth \theta_3 \cosh \theta_2 + Z_{02} \sinh \theta_2}. \quad (8)$$

Since the sum of admittances at y_2 must equal zero,

$$Y_{1-2} + jB_c + Y_{4-2} = 0. \quad (9)$$

Substituting Eqs. (3) and (8) into Eq. (9) yields

$$Y_{01} \coth \theta_1 + jB_c + Y_{02} \frac{Z_{03} \coth \theta_3 \sinh \theta_2 + Z_{02} \cosh \theta_2}{Z_{03} \coth \theta_3 \cosh \theta_2 + Z_{02} \sinh \theta_2} = 0 \quad (10)$$

or

$$\coth \theta_1 + j \frac{B_c}{Y_{01}} + \frac{Y_{02}}{Y_{01}} \frac{\coth \theta_3 \sinh \theta_2 + \frac{Z_{02}}{Z_{03}} \cosh \theta_2}{\coth \theta_3 \cosh \theta_2 + \frac{Z_{02}}{Z_{03}} \sinh \theta_2} = 0. \quad (11)$$

Since region 1 and region 2 have the same propagation constant, $\gamma_{y1} = \gamma_{y2}$, the impedances are proportional to the heights:

$$\frac{Z_{02}}{Z_{01}} = \frac{Y_{01}}{Y_{02}} = \frac{d}{b}. \quad (12)$$

Regions 2 and 3 have equal heights, and since the transverse wave is *TE*, the impedance ratio is

$$\frac{Z_{02}}{Z_{03}} = \frac{\gamma_{y3}}{\gamma_{y2}}. \quad (13)$$

The left side of Eq. (11) may be rewritten as a single fraction. All terms in the denominator are finite, so the numerator may be equated to zero. The resultant expression is

$$\left[\frac{b}{d} \sinh \theta_1 \right] [\gamma_{y2} \cosh \theta_3 \sinh \theta_2 + \gamma_{y3} \sinh \theta_3 \cosh \theta_2] + \left[\cosh \theta_1 + j \frac{B_c}{Y_{01}} \sinh \theta_1 \right] \times [\gamma_{y2} \cosh \theta_3 \cosh \theta_2 + \gamma_{y3} \sinh \theta_3 \sinh \theta_2] = 0. \quad (14)$$

Within each region,

$$\gamma_{xi}^2 + \gamma_{yi}^2 + \gamma_{zi}^2 = -\omega^2 \mu_0 \epsilon_i \quad (15)$$

where

$$\begin{aligned} \epsilon_i &= \epsilon_0 \text{ for } i = 1, 2 \\ &= \epsilon_r \epsilon_0 \text{ for } i = 3. \end{aligned}$$

For *TE* modes, $\gamma_{xi} = 0$ for all regions and $\gamma_{zi} = j\beta$ for all regions; β is the longitudinal propagation constant (above cutoff) for the waveguide configuration. Substituting

$$\begin{aligned} \gamma_{yi} &= \sqrt{\beta^2 - \omega^2 \mu_0 \epsilon_i} \text{ for } \omega^2 \mu_0 \epsilon_i < \beta^2 \\ &= j\sqrt{\omega^2 \mu_0 \epsilon_i - \beta^2} \text{ for } \omega^2 \mu_0 \epsilon_i \geq \beta^2 \end{aligned} \quad (16)$$

and

$$\theta_i = \gamma_{yi} w_i$$

with

$$\begin{aligned} w_1 &= 1/2 (a - s) \\ w_2 &= 1/2 (s - t) \\ w_3 &= 1/2 t \end{aligned} \quad (17)$$

into Eq. (14) yields the transcendental equation in β that must be solved for TE_{n0} (n odd) modes. The smallest root of Eq. (14) is the TE_{10} solution, the next root the TE_{30} solution, etc.

For TE_{n0} (n even) modes, the analysis starts with the equivalent transmission line circuit of Fig. 3b and proceeds in a manner similar to the case for n odd. The resultant transcendental equation is

$$\left[\frac{b}{d} \sinh \theta_1 \right] [\gamma_{y2} \sinh \theta_3 \sinh \theta_2 + \gamma_{y3} \cosh \theta_3 \cosh \theta_2] + \left[\cosh \theta_1 + j \frac{B_c}{Y_{01}} \sinh \theta_1 \right] \times [\gamma_{y2} \sinh \theta_3 \cosh \theta_2 + \gamma_{y3} \cosh \theta_3 \sinh \theta_2] = 0 \quad (18)$$

with Eqs. (16) and (17) being applicable.

If $\epsilon_r = 1$, it is straightforward to show that Eqs. (14) and (18) reduce to the expressions for the odd and even mode cutoff frequencies, respectively, for double-ridged waveguide [1,2]. Also, if $b = d$, B_c equals zero and Eqs. (14) and (18) result in expressions for the odd- and even-mode propagation constants of dielectric slab center loaded rectangular waveguide identical to those obtained by use of ABCD matrices [6].

The discontinuity-susceptance term B_c/Y_{01} is obtained from the *Waveguide Handbook* [8]. Appendix A gives the necessary equations for calculating B_c/Y_{01} in terms of the waveguide dimensions (from Fig. 2) and the effective wavelength λ_g . Note that λ_g is the wavelength of the wave component which is incident normal to the height change. Therefore λ_g of Appendix A is the wavelength of the transverse wave in regions 1 and 2, namely λ_{y1} .

For standard (i.e. air filled) double-ridged waveguide $\epsilon_r = 1$; thus

$$\gamma_{y1} = \gamma_{y2} = \gamma_{y3} = j\beta_y$$

and $\lambda_{yi} = 2\pi/\beta_y$ is a real constant for a given configuration. However, for the general case $\epsilon_r > 1$ and

$$\gamma_{y1} = \gamma_{y2} \neq \gamma_{y3},$$

with the result that the values of γ_{yi} that satisfy Eq. (14) or (18), subject to (16) and (17), are no longer constant but depend on the frequency. This is of course to be expected, since for any nonhomogeneous waveguide a $1/\sqrt{1 - (f_{c/f})^2}$ term no longer describes the dispersive nature. However, there is another problem because of the inhomogeneity. At all frequencies above cutoff, the transverse wave propagation constant in the dielectric region will be entirely imaginary; i.e.,

$$\gamma_{y3} = j\beta_{y3} \text{ for } \omega > \omega_c.$$

However, there is a critical frequency ω_{crit} (ω_{crit} is greater than the cutoff frequency ω_c , how much greater depends upon the degree of dielectric loading) such that for frequencies greater than ω_{crit} the transverse propagation constant in regions 1 and 2 is real, that is,

$$\gamma_{y1} = \gamma_{y2} = \alpha_{y1} \text{ for } \omega > \omega_{crit}.$$

When $\omega > \omega_{crit}$, the transverse "wave" in these regions is no longer a resonant traveling wave but rather the fields are decaying exponentially away from the dielectric region, and the concept of wavelength in the region of the discontinuity is not meaningful. The expression for the B_c/Y_{01} term from Ref. 8 is no longer applicable; indeed, the validity of the equivalent transmission line circuit for the waveguide height change (a shunt susceptance at the junction of two transmission lines of unequal characteristic impedance) is questionable for operation below cutoff. Also, the calculation for B_c/Y_{01} is based on a model which assumes that the waveguide extends to infinity in both directions away from the height discontinuity; in practice, the assumption is valid if additional mismatches are far enough removed from the height discontinuity so that the local fields have decayed to small proportions. These local fields are the evanescent modes of the fringing fields caused by the height discontinuity, and they decay very rapidly.

Future investigation is planned to model an equivalent circuit of the waveguide height change to include operation below as well as above the cut-off frequency, and to include the proximity effects of waveguide walls and dielectric center loading. However, for this report the following two engineering assumptions are made:

1. The B_c/Y_{01} term can be neglected for frequencies below the critical frequency. Since

$$\frac{B_c}{Y_{01}} \rightarrow 0 \text{ as } \omega \rightarrow \omega_{crit}^{(+)}$$

and for $\omega < \omega_{crit}$ the fields of the transverse wave are decaying exponentially in the region of the height discontinuity, a small shunt susceptance term will have only a minor effect on the solution for β . Equations (14) and (18) are transcendental equations and must be solved by some algorithm using trial values of β . If a trial value of β yields an imaginary transverse propagation constant in region 1, the B_c/Y_{01} term is calculated with

$$\gamma_{y1} = j\beta_{y1} \text{ and } \lambda_{y1} = \frac{2\pi}{\beta_{y1}}.$$

If the trial value of β results in γ_{y1} real, the B_c/Y_{01} term is neglected, i.e. set equal to zero, in Eqs. (14) and (18).

2. Proximity effects can be neglected in the calculation of B_c/Y_{01} .

Although the validity of these two assumptions may be questioned from a rigorous theoretical aspect, the close agreement between calculated and measured values of β for different configurations (shown in Figs. 4 and 5) indicates that both assumptions result in accuracy sufficient for most practical applications.

A listing of a computer program to solve for the principal (TE_{10}) mode propagation constant of dielectric slab center loaded double-ridged waveguide is given in Appendix B.

All discussions and calculations thus far have assumed a double-ridged waveguide configuration. For the asymmetric or single-ridged waveguide configurations shown in Fig. 6, Eqs. (14) and (18) remain valid; however, the expression for B_c/Y_{01} must have λ_{y1} replaced by $1/2 \lambda_{y1}$.

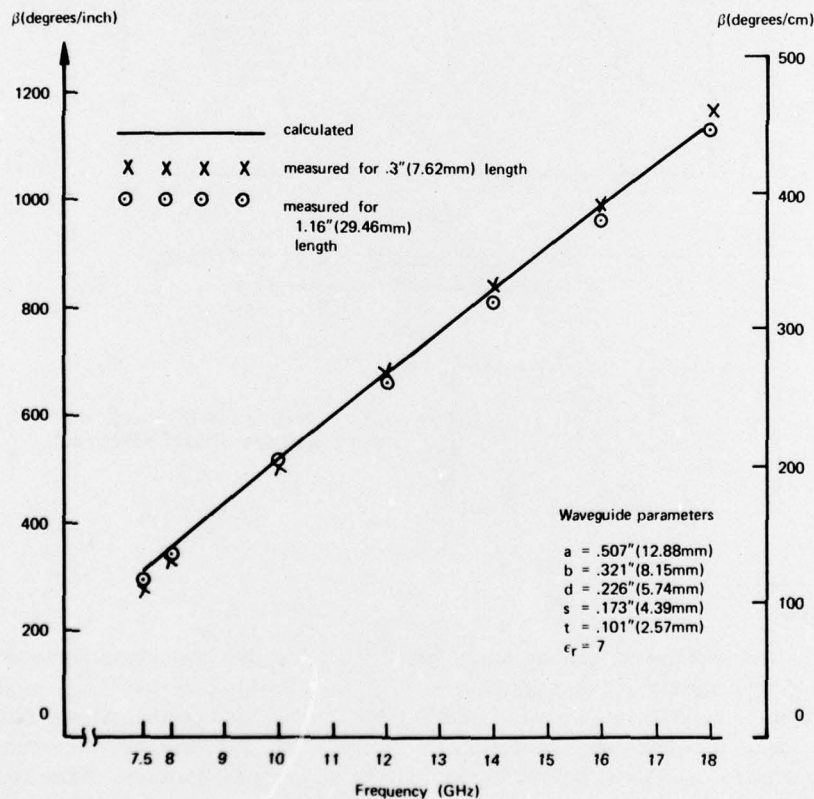


Fig. 4 — Calculated and measured values of β for dielectric slab loaded double-ridged waveguide

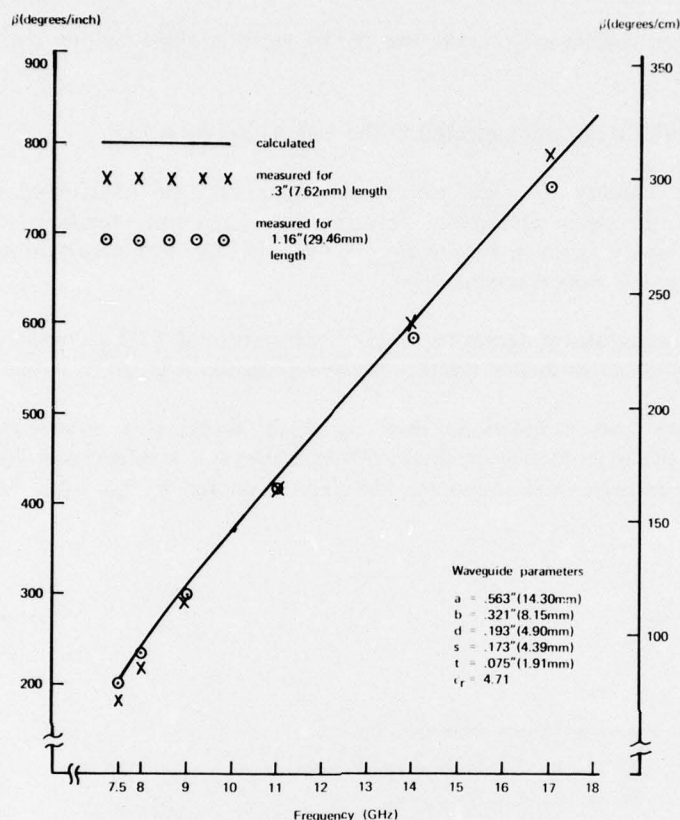


Fig. 5 — Calculated and measured values of β for dielectric slab loaded doubled-ridged waveguide

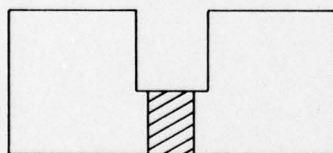


Fig. 6 — Cross section of dielectric slab center loaded single-ridged waveguide

CONCLUSION

Based on the equivalent transmission line circuit for the transverse component of the propagating electromagnetic wave, expressions have been derived for the TE_{n0} mode propagation constants of a dielectric slab center loaded ridged waveguide configuration. These expressions are transcendental equations involving the propagation constant, but they can readily be solved with a computer. Based on the agreement between calculated and measured data, certain assumptions made in the derivation appear valid. The analysis should prove useful in designing transformers to match ridged waveguide to dielectric or ferrite slab center loaded rectangular waveguide.

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Appendix A EQUIVALENT CIRCUIT FOR A CHANGE IN HEIGHT OF RECTANGULAR WAVEGUIDE

For a height change of rectangular waveguide as shown in Figs. A1a and A1b, the equivalent circuit given by the *Waveguide Handbook** is shown in Fig. A1c. The characteristic admittances of the different height waveguides are Y_0 and Y_0' , T is the effective terminal plane, B_c is the effective shunt capacitive susceptance, and λ_g is the wavelength of the propagating wave. The admittance ratio is

$$\frac{Y_0}{Y_0'} = \frac{d}{b} = \alpha$$

and at the terminal plane T

$$\frac{B_c}{Y_0} = \frac{2b}{\lambda_g} \left\{ \ln \left(\frac{1 - \alpha^2}{4\alpha} \right) + \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right) \ln \left(\frac{1 + \alpha}{1 - \alpha} \right) + 2 \frac{A + A' + 2C}{AA' - C^2} \right. \\ \left. + \left(\frac{b}{4\lambda_g} \right)^2 \left(\frac{1 - \alpha}{1 + \alpha} \right)^{4\alpha} \left(\frac{5\alpha^2 - 1}{1 - \alpha^2} + \frac{4}{3} \frac{\alpha^2 C}{A} \right) \right\}$$

where

$$A = \left(\frac{1 + \alpha}{1 - \alpha} \right)^{2\alpha} \frac{1 + \sqrt{1 - \left(\frac{b}{\lambda_g} \right)^2}}{1 - \sqrt{1 - \left(\frac{b}{\lambda_g} \right)^2}} - \frac{1 + 3\alpha^2}{1 - \alpha^2}$$

$$A' = \left(\frac{1 + \alpha}{1 - \alpha} \right)^{2/\alpha} \frac{1 + \sqrt{1 - \left(\frac{d}{\lambda_g} \right)^2}}{1 - \sqrt{1 - \left(\frac{d}{\lambda_g} \right)^2}} + \frac{3 + \alpha^2}{1 - \alpha^2}$$

and

$$C = \left(\frac{4\alpha}{1 - \alpha^2} \right)^2$$

The equivalent circuit is valid for $b/\lambda_g < 1$.

*N. Marcuvitz, *Waveguide Handbook*, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1951.

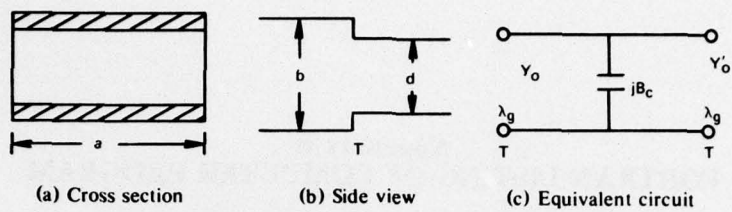


Fig. A-1 — Height change of rectangular waveguide and equivalent circuit

Appendix B FORTRAN LISTING OF COMPUTER PROGRAM

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00100  C THIS IS PROGRAM DRMGDL.FOR - CWY -OCT 75
00200      INTEGER RIK
00300      REAL KXAIR
00400      PI=3.1415927
00500      C=2.997925E+08
00600      R1=39.37008
00700      R2=2.0*R1
00800      RRMDI=180.0/(PI*R1)
00900      C1=(2.0E+09*PI/C)**2
01000      NEWRUN=0
01100      TYPE 600
01200  600      FORMAT (//// PROGRAM DRMGDL/CWY/OCT 75/// COMPUTES
01300      1 TE10 CUTOFF FREQUENCIES AND PROPAGATION CONSTANTS OF//
01400      2 / SYMMETRIC DIELECTRIC LOADED DOUBLE RIDGED WAVEGUIDE/)
01500  105      TYPE 605
01600  605      FORMAT(/// WAVEGUIDE DIMENSIONS IN INCHES - A,B,D,S: '$)
01700      READ(5,*)A,B,D,S
01800  106      TYPE 615
01900  615      FORMAT (// RELATIVE DIELECTRIC CONSTANT OF CENTER
02000      1 LOADING = '$)
02100      READ(5,*)EPSR
02200      TYPE 625
02300  625      FORMAT (/ WIDTH IN INCHES OF CENTER LOADING = '$)
02400      ACCEPT 630,T
02500  630      FORMAT (F8.3)
02600      IF(T.LT.S)GO TO 108
02700      TYPE 631
02800  631      FORMAT (/ DIELECTRIC WIDTH MUST BE LESS THAN
02900      1 RIDGE WIDTH ---- TRY AGAIN/)
03000      GO TO 105
03100  108      TYPE 606
03200  606      FORMAT (/// DRMGDL PARAMETERS ----- DIMENSIONS IN
03300      1 INCHES//8X/ A/9X/B/9X/D/9X/S/12X/T/6X4HEPS//1X$)
03400      TYPE 607,A,B,D,S,T,EPSR
03500  607      FORMAT (4F10.4,F13.4,F10.3)
03600      R=D/B
03700      RS=R**2
03800      IFR=0
03900      IF(ABS(R-1.0).LT.1.0E-06)IFR=1
04000      W1=(A-S)/R2
04100      W2=(S-T)/R2
04200      W3=T/R2
04300      CEREST=1.+(1./R-1.)*COS(PI*(A-S)/(2.*A))
04400      CLREST=CEREST+(EPSR-1.)/R*COS(PI*(A-T)/(2.*A))
04500      EDCTRY=CLREST/CEREST
04600      ALCEST=A*CLREST*(R+(1.-R)*SIN(PI*(A-S)/(2.*A)))
04700  C THE ABOVE FOUR QUANTITIES ARE TO BE USED FOR CALCULATING
04800  C APPROXIMATE (STARTING VALUES) OF CUTOFF FREQUENCIES AND
04900  C PROPAGATION CONSTANTS
05000      IBC=1

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05100      FREQ=C*R1/(ALCEST*2.0E+09)
05200      DELBY=0.31*FREQ
05300      BY=0.0
05400      GO TO 112
05500  109    IF(NEWRUN.LT.2)GO TO 210
05600      IF(FSTART.GT.FCGHZ)GO TO 220
05700  210    TYPE 635
05800  635    FORMAT (// FREQUENCIES IN GHZ - START,STOP,INCREMENT: '$)
05900      READ(5,*)FSTART,FSTOP,DELF
06000  640    FORMAT (F9.3,1X,F9.3,1X,F9.3)
06100  220    IF(FSTART.LT.1.0E-13)GO TO 180
06200      IF(FSTART.GT.FCGHZ)GO TO 230
06300      TYPE 645
06400  645    FORMAT(' FREQUENCY MUST BE GREATER THAN CUTOFF')
06500      GO TO 210
06600      IF(FSTOP.LT.1.0E-13)FSTOP=FSTART-1.0
06700  230    TYPE 655
06800  655    FORMAT (/4X4HFREQ08X4HBETA9X3HGWL7X5HRATIO8X5HKXAIR/
06900  1 5X3HGHZ6X6HDEG/IN6X6HINCHES4X8HGWL/FSWL7X6HR OR I/)
07000  110    IFREQ=0
07100      FREQ=FSTART
07200  111    IFREQ=IFREQ+1
07300      BY=PI*2.E+09/C*SQRT(EDCTRY*(FREQ**2+FCGHZ**2))
07400  C THIS IS A FIRST TRY FOR BETA
07500      DELBY=-0.31*BY
07600  112    ICROSS=0
07700      ITAN=0
07800      IBTRY=0
07900  115    C1F=C1*FREQ**2
08000      C1FEP=C1F*EPSR
08100  120    IBTRY=IBTRY+1
08200      IF(IBTRY.LT.26)GO TO 122
08300      TYPE 705
08400  705    FORMAT (' MORE THAN 25 TRIES AT ROOT')
08500      GO TO 170
08600  122    BYSQ=BY**2
08700      GX3SQ=C1FEP-BYSQ
08800      GX2SQ=C1F-BYSQ
08900      GX3=SQRT(ABS(GX3SQ))
09000      GX2=SQRT(ABS(GX2SQ))
09100      IF(GX3SQ)130,132,132
09200  130    CHS3=SINH(GX3*M3)
09300      CHC3=COSH(GX3*M3)
09400      IRGX3=1
09500      GO TO 134
09600  132    CHS3=SIN(GX3*M3)
09700      CHC3=COS(GX3*M3)
09800      IRGX3=-1
09900  134    CONTINUE
10000      IF(GX2SQ)136,138,138

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10100 136 CHS2=SINH(GX2*W2)
10200 CHC2=COSH(GX2*W2)
10300 CHS1=SINH(GX2*W1)
10400 CHC1=COSH(GX2*W1)
10500 KXAIR=GX2
10600 RIK=1HR
10700 IRGX2=1
10800 GO TO 140
10900 138 CHS2=SIN(GX2*W2)
11000 CHC2=COS(GX2*W2)
11100 CHS1=SIN(GX2*W1)
11200 CHC1=COS(GX2*W1)
11300 IRGX2=-1
11400 KXAIR=GX2*PRMDI
11500 RIK=1HI
11600 140 BOY=0.0
11700 IF(IFR.EQ.1) GO TO 153
11800 IF(IRGX2.EQ.1) GO TO 153
11900 C CALCULATE B/Y TERM
12000 P=(1+R)/(1-R)
12100 GL=2.0*PI/GX2
12200 P2=SQRT(1.0-(B/(R1*GL))**2)
12300 P3=SQRT(1.0-(D/(R1*GL))**2)
12400 PA=P**2*(2.0/R)+(1.0+P2)/(1.0-P2)-(1.0+3.0*RS)/(1.0-RS)
12500 PAP=P**2*(2.0/R)+(1.0+P3)/(1.0-P3)+(3.0*RS)/(1.0-RS)
12600 PC=((4.0/R)/(1.0-RS))**2
12700 PT1=ALOG((1.0-RS)/(4.0/R)+P**2*(0.5*(R+1.0/R)))
12800 PT2=2.0*(PA+PAP+2.0*PC)/(PA+PAP-PC**2)
12900 PT3=(B/(R1*4.0*GL))**2*(1.0/P)**2*(4.0/R)+(5.0*RS
13000 1 -1.0)/(1.0-RS)+4.0*RS*PC/(3.0*PA)**2
13100 BOY=2.0*B*(PT1+PT2+PT3)/(R1*GL)
13200 C CALCULATE F(BETA)
13300 153 FBETA=R*(-BOY+CHS1+CHC1)+(GX2/GX3*CHC3+CHC2
13400 1 +IRGX3*CHS3+CHS2)+IRGX2*CHS1+(GX2/GX3*CHC3+CHS2
13500 2 +IRGX2*IRGX3*CHS3+CHC2)
13600 IF(IBC.EQ.1) BY=FREQ
13700 C ROOT SEARCH ROUTINE
13800 IF(ABS(FBETA).LT.1.0E-08) GO TO 170
13900 IF(IBTRY.EQ.1) GO TO 163
14000 IF(ITAN.EQ.1) GO TO 164
14100 IF(FBETA*FBOLD.LT.0.0) GO TO 161
14200 IF(ABS(FBETA).GT.ABS(FBOLD)) DELBY=-DELBY
14300 GO TO 162
14400 161 DELBY=-DELBY
14500 ICROSS=1
14600 162 IF(ICROSS.EQ.1) DELBY=0.5*DELBY
14700 163 BYNEW=BY+DELBY
14800 IF(ABS((BY-BYNEW)/BY).LT.0.1) ITAN=1
14900 GO TO 166
15000 164 IF(ABS(BY-BYOLD).LT.1.E-05.AND.FBETA.LT.1.E-06) GO TO 170

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15100      BYNEW=BY-FBETA*(BY-BYOLD)/(FBETA-FBOLD)
15200      166      BYOLD=BY
15300      BY=BYNEW
15400      FBOLD=FBETA
15500      IF (IBC.EQ.2) GO TO 120
15600      FREQ=BY
15700      BY=0.0
15800      GO TO 115
15900      170      IF (IBC.EQ.2) GO TO 175
16000      FCGHZ=BY
16100      TYPE 658,FCGHZ,BOY
16200      658      FORMAT (// TE10 MODE CUTOFF FREQUENCY IN GHZ = 'F7.4'
16300      1      B/Y = 'F7.3')
16400      IBC=2
16500      GO TO 109
16600      173      CONTINUE
16700      175      BYDI=BY*RRMDI
16800      GML=360.0/BYDI
16900      FSWL=R1*C/(FREQ*1.0E+09)
17000      RGLFS=GML/FSWL
17100      177      TYPE 660,FREQ,BYDI,GML,RGLFS,KXAIR,RIK
17200      660      FORMAT (1X,F7.3,3X,F9.2,3X,F9.4,4X,F8.4,3X,F8.2,1X,A1)
17300      IF (FREQ.GE.FSTOP) GO TO 180
17400      FREQ=FREQ+DELF
17500      GO TO 111
17600      180      TYPE 665
17700      665      FORMAT (/// WISH NEW PARAMETERS? NONE=0, ALL=1,
17800      1      CENTER LOADING=2, FREQ=3      '$')
17900      ACCEPT 670,NEWRUN
18000      670      FORMAT (I1)
18100      GO TO (199,105,106,210,180)NEWRUN+1
18200      199      CONTINUE
18300      END

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